Digital Signal Processing

Lab 07: Fourier Analysis for CT Signals and Systems

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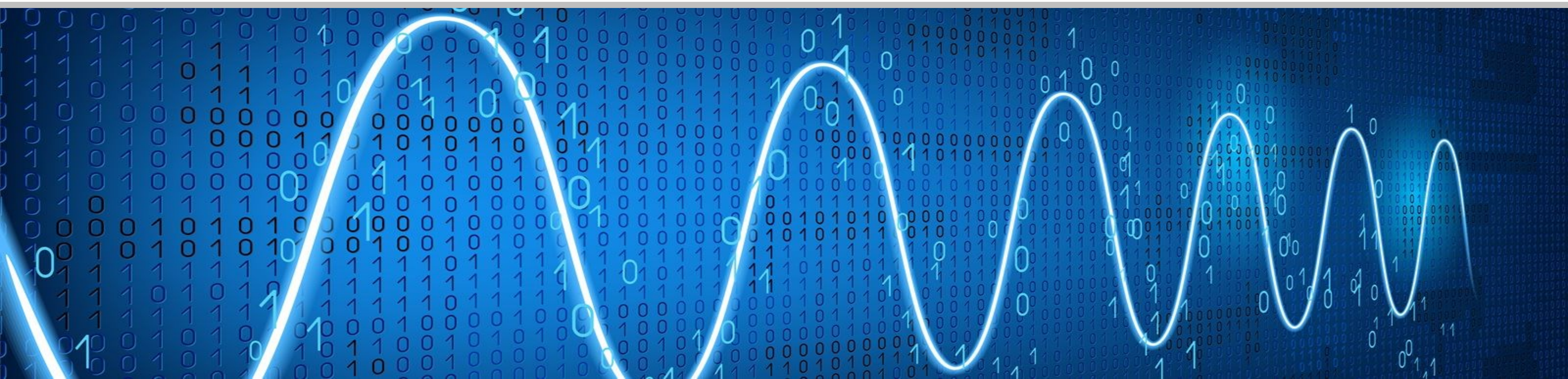


Fourier Analysis for Continuous-Time Signals and Systems

The purpose of this lab is to

- Learn techniques for representing continuous-time periodic signals using orthogonal sets of periodic basis functions.
- Study properties of exponential, trigonometric and compact Fourier series.

Continuous-Time Signals and Systems

- In Chapters 1 and 2 we have developed techniques for analyzing continuous-time signals and systems from a time-domain perspective.
- A continuous-time signal can be modeled as a function of time.

$$y(t) = \text{Sys}\{x(t)\}$$

- A CTLTI system can be represented by a constant-coefficient linear differential equation, or by means of an impulse response.
- The output signal of a CTLTI system can be determined by solving the differential equation or by using the convolution operation.

Analysis of Periodic Continuous-Time Signals

- Most periodic continuous-time signals in engineering problems can be expressed as linear combinations of sinusoidal basis functions.
- The basis functions can either be individual sine and cosine functions, or they can be in the form of complex exponential functions that combine sine and cosine functions together.

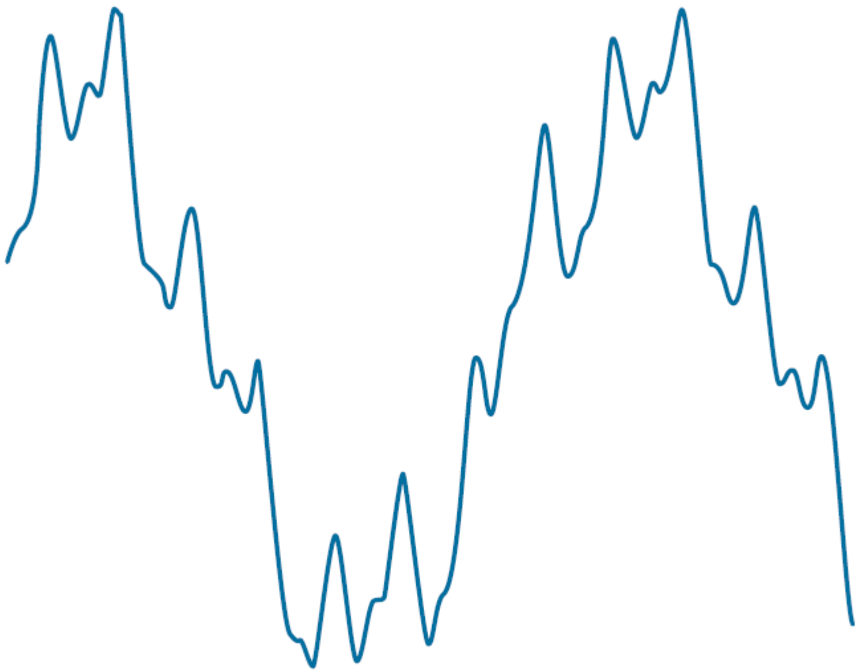
$$x(t) = A\sin(\omega_0 t + \theta)$$

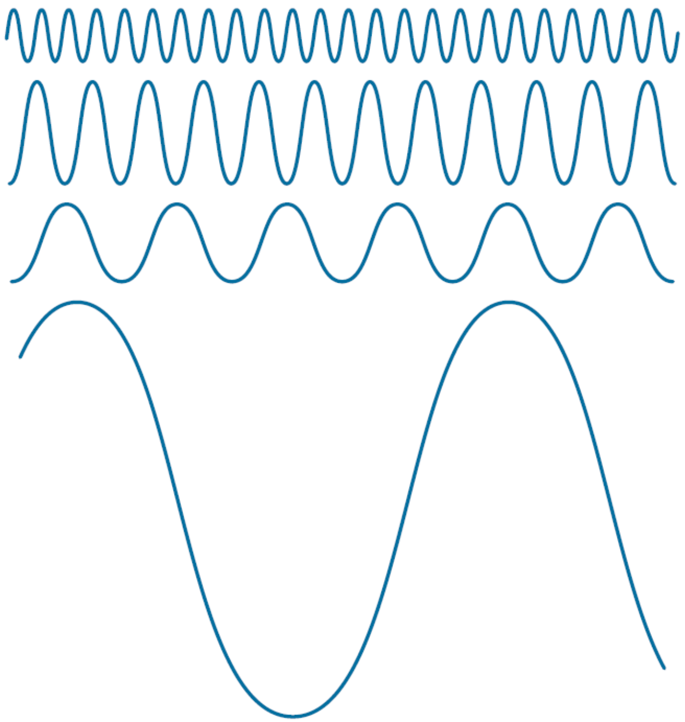
$$x(t) = A\cos(\omega_0 t + \theta)$$

$$e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$

Fourier Series

- **Fourier Series**: Any periodic function can be expressed as the sum of sines and/or cosines of different frequencies, each multiplied by a different coefficient.





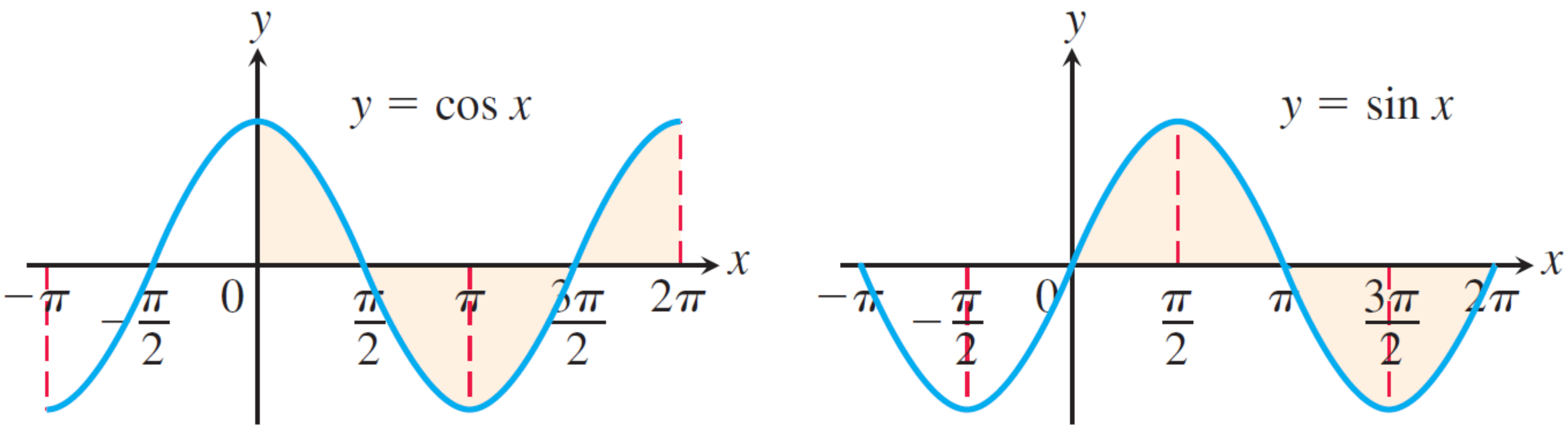
Approximating a Periodic Signal With Trigonometric Functions

- A signal $\tilde{x}(t)$ which is periodic with period T_0 has the property

$$\tilde{x}(t) = \tilde{x}(t + T_0)$$

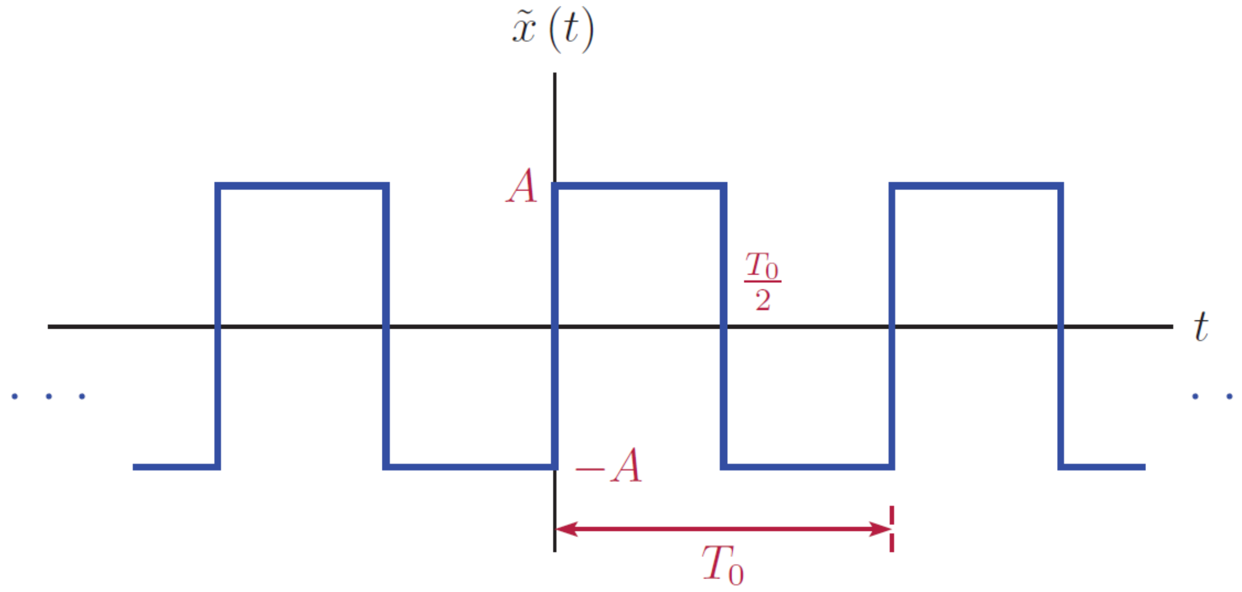
- Furthermore, a signal that is periodic with period T_0 is also periodic with kT_0 for any integer k.

$$\tilde{x}(t) = \tilde{x}(t + kT_0)$$



Approximating a Periodic Signal With Trigonometric Functions

- Suppose that we wish to approximate this signal using just one trigonometric function.



- The first two questions that need to be answered are:

1. Should we use a sine or a cosine?

2. How should we adjust the parameters of the trig function?

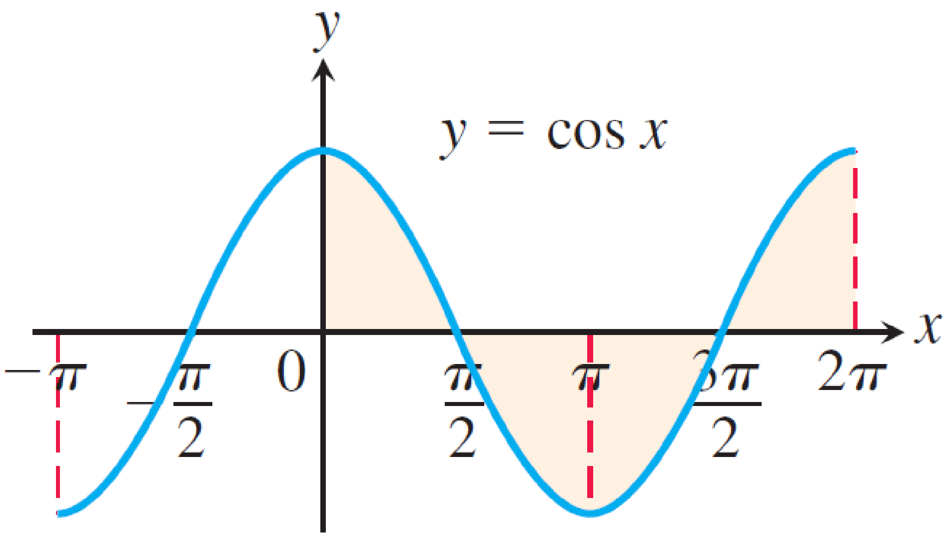
Approximating a Periodic Signal With Trigonometric Functions

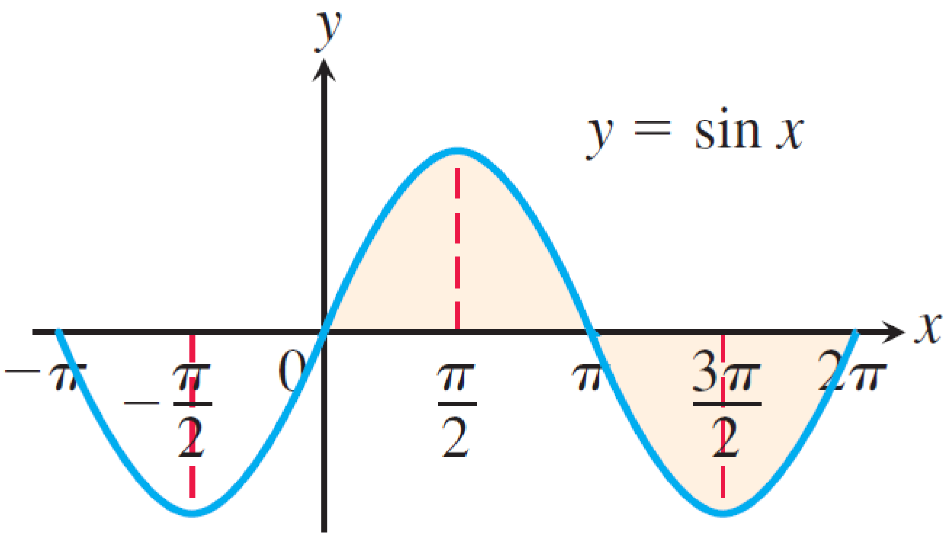
- The sine function has odd symmetry, since

$$\sin(-x) = -\sin(x)$$

- On the other hand, the cosine function has even symmetry since

$$\cos(-x) = \cos(x)$$

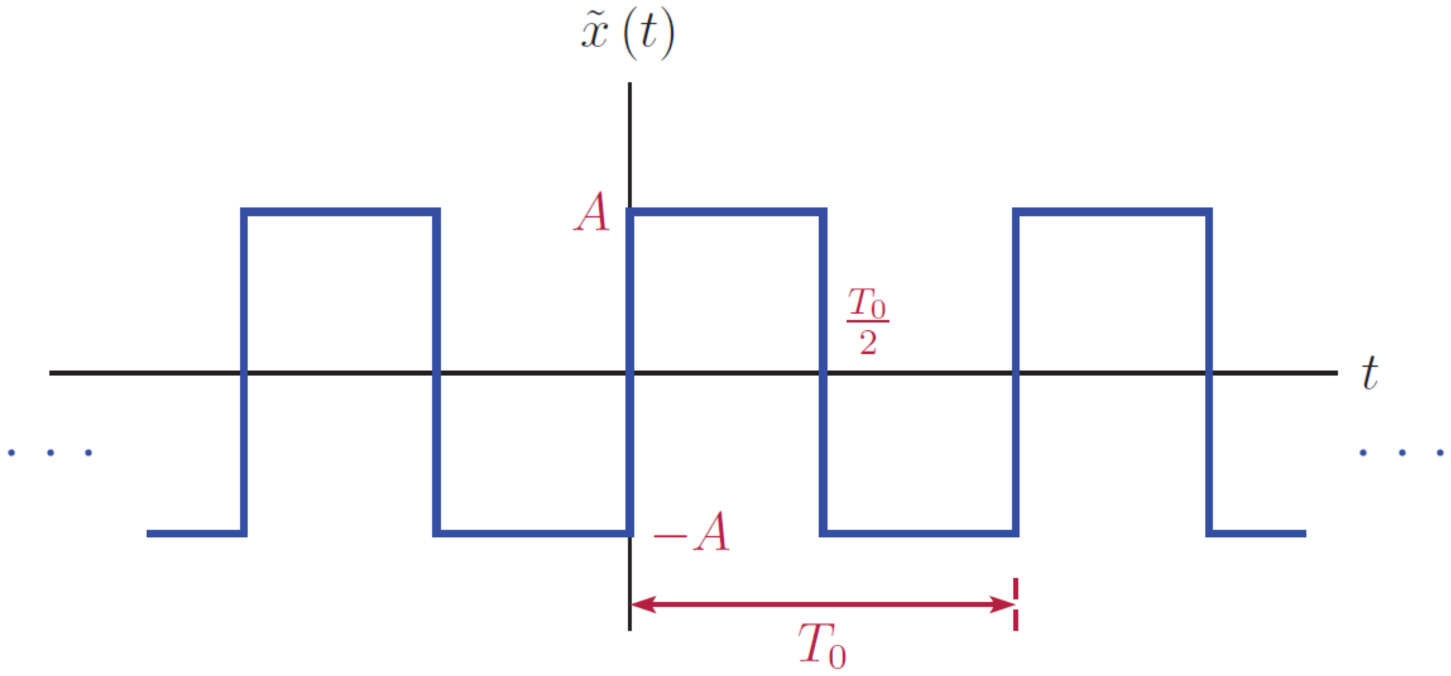




Approximating a Periodic Signal With Trigonometric Functions

- Therefore it would make sense to choose the sine function.
- Our approximation would be in the form

$$\tilde{x}(t) \approx b_1 \sin(\omega t)$$



Approximating a Periodic Signal With Trigonometric Functions

- Since $\tilde{x}(t)$ has a fundamental period of T_0, it would make sense to pick a sine function with the same fundamental period.

$$\omega = \frac{2\pi}{T_0} = \omega_0 = 2\pi f_0$$

$$\tilde{x}^{(1)}(t) = b_1 \sin(\omega_0 t)$$

- Our next task is to determine the value of the coefficient b_1.
- How should b_1 be chosen to get the best approximation?

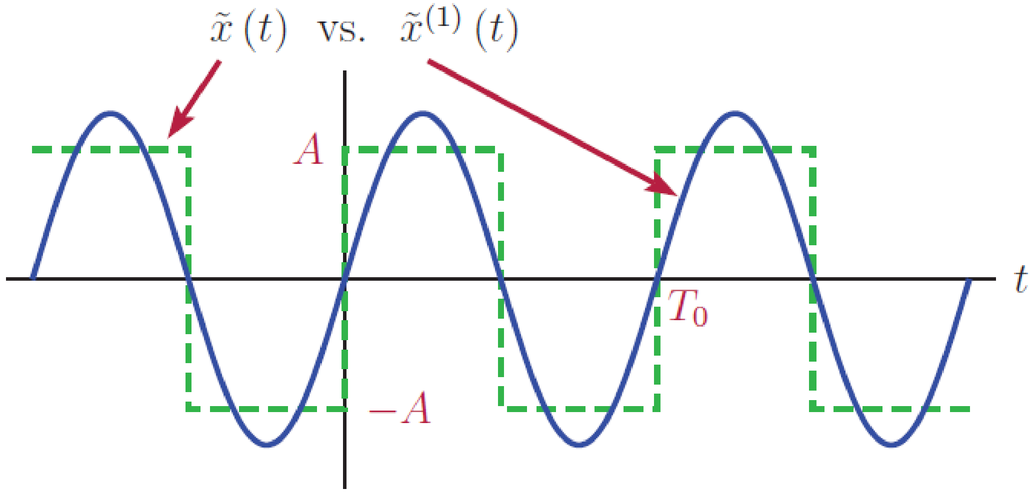
Approximating a Periodic Signal With Trigonometric Functions

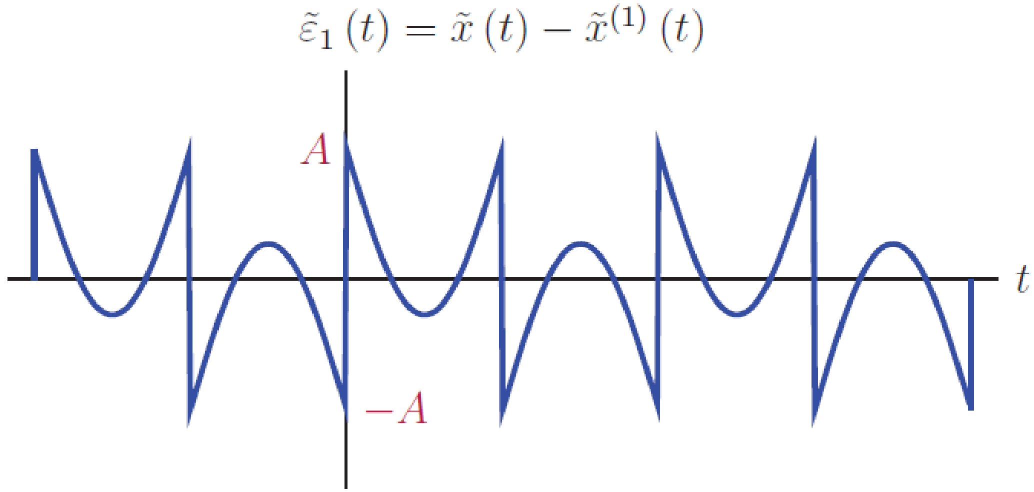
- Let us define the approximation error as the difference between the square-wave signal and its approximation:

$$\tilde{\varepsilon}_1(t) = \tilde{x}(t) - \tilde{x}^{(1)}(t) = \tilde{x}(t) - b_1 \sin(\omega_0 t)$$

- The best approximation to using only one trigonometric function is

$$\tilde{x}^{(1)}(t) = \frac{4A}{\pi} \sin(\omega_0 t)$$





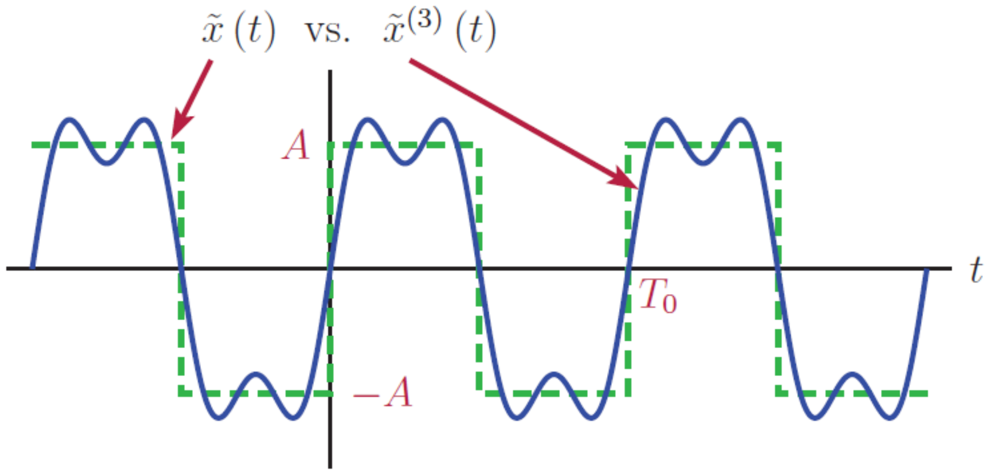
Approximating a Periodic Signal With Trigonometric Functions

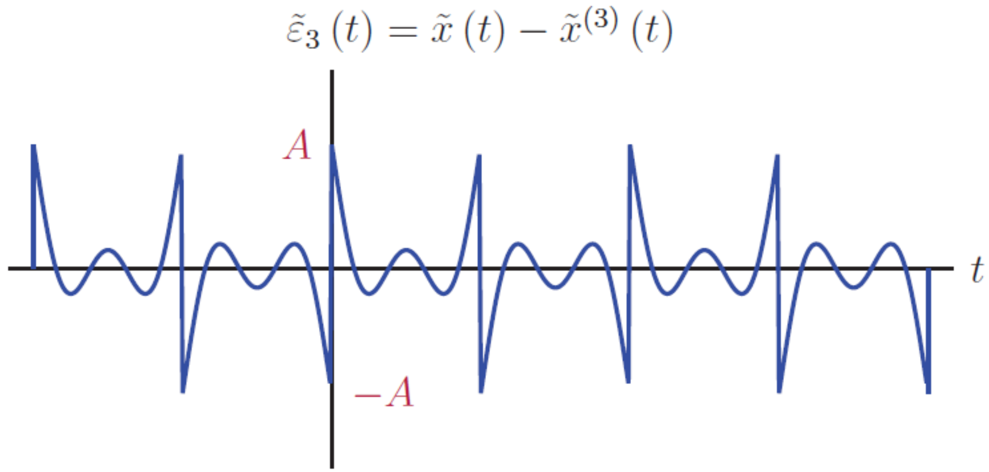
- The three-frequency approximation

$$\tilde{x}^{(3)}(t) = b_1 \sin(\omega_0 t) + b_2 \sin(2\omega_0 t) + b_3 \sin(3\omega_0 t)$$

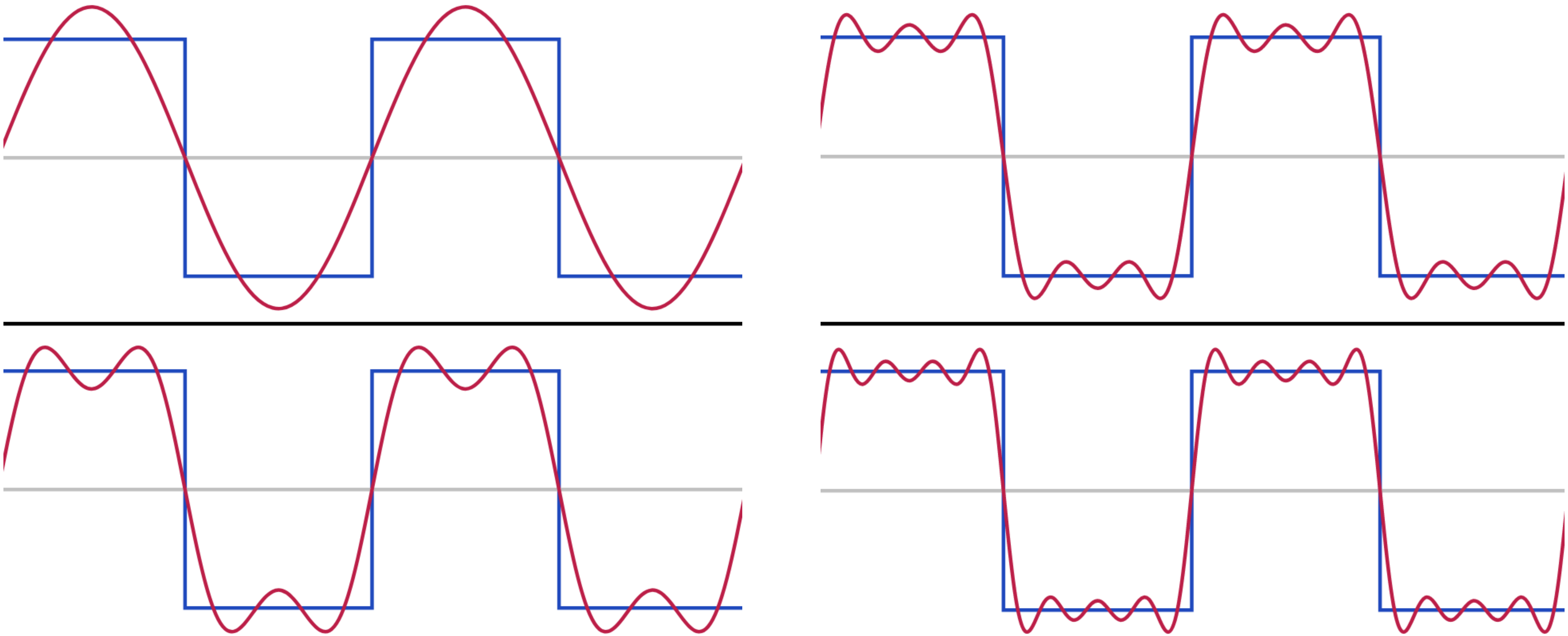
- The optimum coefficient values

$$b_1 = \frac{4A}{\pi}, \quad b_2 = 0, \quad \text{and} \quad b_3 = \frac{4A}{3\pi}$$





Trigonometric Fourier Series (TFS)



Trigonometric Fourier Series (TFS)

- Consider a signal $\tilde{x}(t)$ that is periodic with fundamental period T_0 and associated fundamental frequency $f_0 = 1/T_0$.
- We want to represent this signal using a linear combination of sinusoidal functions in the form

$$\tilde{x}(t) = a_0 + a_1 \cos(\omega_0 t) + a_2 \cos(2\omega_0 t) + \ldots + a_k \cos(k\omega_0 t) \ldots$$
$$+ b_1 \sin(\omega_0 t) + b_2 \sin(2\omega_0 t) + \ldots + b_k \sin(k\omega_0 t) + \ldots$$

- Using more compact notation

$$\tilde{x}(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t)$$

Trigonometric Fourier Series (TFS)

Trigonometric Fourier series (TFS):

1. Synthesis equation:

$$\tilde{x}(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t)$$

2. Analysis equations:

$$a_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} \tilde{x}(t) \cos(k\omega_0 t)\, dt\,, \quad \text{for} \quad k = 1, \ldots, \infty$$

$$b_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} \tilde{x}(t) \sin(k\omega_0 t)\, dt\,, \quad \text{for} \quad k = 1, \ldots, \infty$$

$$a_0 = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} \tilde{x}(t)\, dt \qquad \text{(dc component)}$$

Integrals

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C$$

$$\int \sin(x)\,dx = -\cos(x) + C$$

$$\int \sin(ax)\,dx = \frac{-1}{a}\cos(ax) + C$$

$$\int e^{x}\,dx = e^{x} + C$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

Integrals

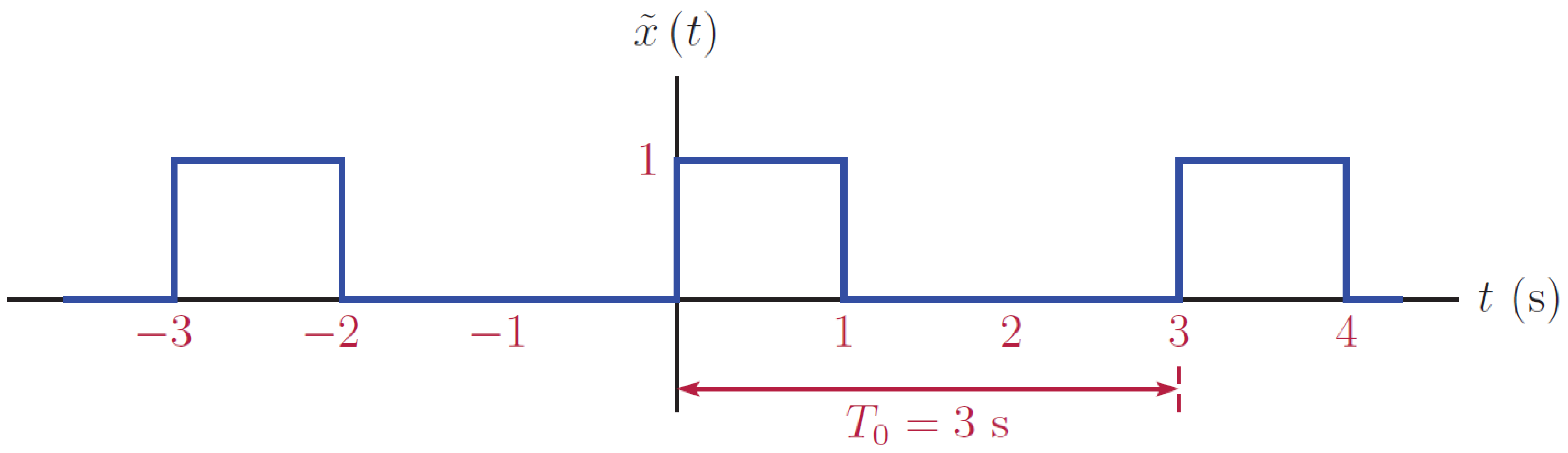
$$\int_{-a}^{a} \cos(x)\,dx = \sin(x)\Big|_{-a}^{a} = \sin(a) - \sin(-a) = \sin(a) + \sin(a) = 2\sin(a)$$

$$\int_{-a}^{a} \sin(x)\,dx = -\cos(x)\Big|_{-a}^{a} = -\cos(a) + \cos(-a) = -\cos(a) + \cos(a) = 0$$

Example 4.1

Example 4.1: Trigonometric Fourier series of a periodic pulse train

A pulse-train signal $\tilde{x}(t)$ with a period of $T_0 = 3$ seconds is shown in Fig. 4.5. Determine the coefficients of the TFS representation of this signal.



Example 4.1 – Solution

Solution: In using the integrals given by Eqns. (4.41), (4.42), and (4.43), we can start at any arbitrary time instant t_0 and integrate over a span of 3 seconds. Applying Eqn. (4.43) with $t_0 = 0$ and $T_0 = 3$ seconds, we have

$$a_0 = \frac{1}{3}\left[\int_0^1 (1)\,dt + \int_1^3 (0)\,dt\right] = \frac{1}{3}$$

The fundamental frequency is $f_0 = 1/T_0 = 1/3$ Hz, and the corresponding value of ω_0 is

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{3} \quad \text{rad/s.}$$

Using Eqn. (4.41), we have

$$\begin{aligned} a_k =& \frac{2}{3}\left[\int_0^1 (1)\,\cos\left(2\pi kt/3\right)\,dt + \int_1^3 (0)\,\cos\left(2\pi kt/3\right)\,dt\right] \\ =& \frac{\sin\left(2\pi k/3\right)}{\pi k}\,, \qquad \text{for } k = 1, 2, \ldots, \infty \end{aligned}$$

Example 4.1 – Solution

Finally, using Eqn. (4.42), we get

$$b_k = \frac{2}{3}\left[\int_0^1 (1)\,\sin\left(2\pi kt/3\right)\,dt + \int_1^3 (0)\,\sin\left(2\pi kt/3\right)\,dt\right]$$

$$= \frac{1-\cos\left(2\pi k/3\right)}{\pi k}\,, \qquad \text{for}\ \ k = 1, 2, \ldots, \infty$$

Using these coefficients in the synthesis equation given by Eqn. (4.40), the signal $x(t)$ can now be expressed in terms of the basis functions as

$$\tilde{x}(t) = \frac{1}{3} + \sum_{k=1}^{\infty}\left(\frac{\sin\left(2\pi k/3\right)}{\pi k}\right)\cos\left(2\pi kt/3\right) + \sum_{k=1}^{\infty}\left(\frac{1-\cos\left(2\pi k/3\right)}{\pi k}\right)\sin\left(2\pi kt/3\right) \qquad (4.44)$$

Example 4.2

Example 4.2: Approximation with a finite number of harmonics

Consider again the signal $\tilde{x}(t)$ of Example 4.1. Based on Eqns. (4.25) and (4.26), it would theoretically take an infinite number of cosine and sine terms to obtain an accurate representation of it. On the other hand, values of coefficients a_k and b_k are inversely proportional to k, indicating that the contributions from the higher order terms in Eqn. (4.44) will decline in significance. As a result we may be able to neglect high order terms and still obtain a reasonable approximation to the pulse train. Approximate the periodic pulse train of Example 4.1 using (a) the first 4 harmonics, and (b) the first 10 harmonics.

Example 4.2 – Solution

Solution: Recall that we obtained the following in Example 4.1:

$$a_0 = \frac{1}{3}\,, \qquad a_k = \frac{\sin(2\pi k/3)}{\pi k}\,, \qquad b_k = \frac{1-\cos(2\pi k/3)}{\pi k}$$

These coefficients have been numerically evaluated for up to $k = 10$, and are shown in Table 4.1.

k	a_k	b_k
0	0.3333	
1	0.2757	0.4775
2	−0.1378	0.2387
3	0.0	0.0
4	0.0689	0.1194
5	−0.0551	0.0955
6	0.0	0.0
7	0.0394	0.0682
8	−0.0345	0.0597
9	0.0	0.0
10	0.0276	0.0477

Table 4.1 – TFS coefficients for the pulse train of Example 4.2.

Example 4.2 – Solution

Let $\tilde{x}^{(m)}(t)$ be an approximation to the signal $\tilde{x}(t)$ utilizing the first m harmonics of the fundamental frequency:

$$\tilde{x}^{(m)}(t) = a_0 + \sum_{k=1}^{m} a_k \cos(k\omega_0 t) + \sum_{k=1}^{m} b_k \sin(k\omega_0 t) \tag{4.45}$$

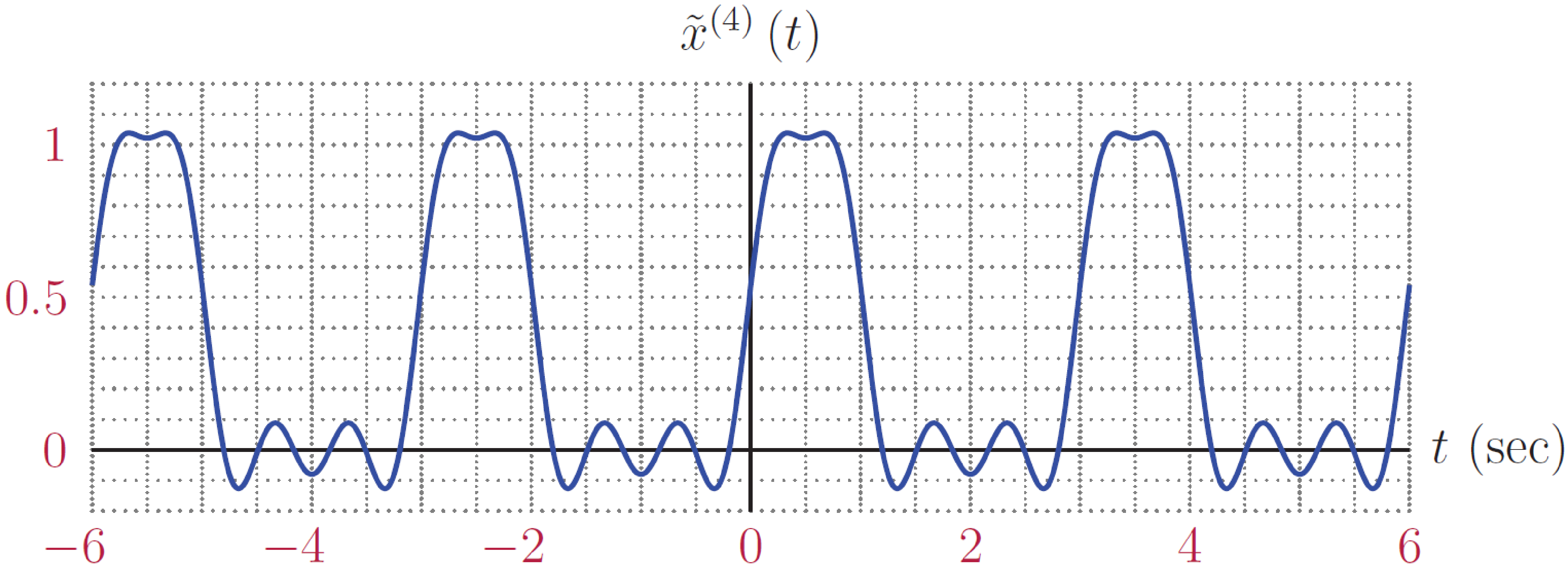
Using $m = 4$, we have

$$\tilde{x}^{(4)}(t) = 0.3333 + 0.2757 \cos(2\pi t/3) - 0.1378 \cos(4\pi t/3) + 0.0689 \cos(8\pi t/3)$$

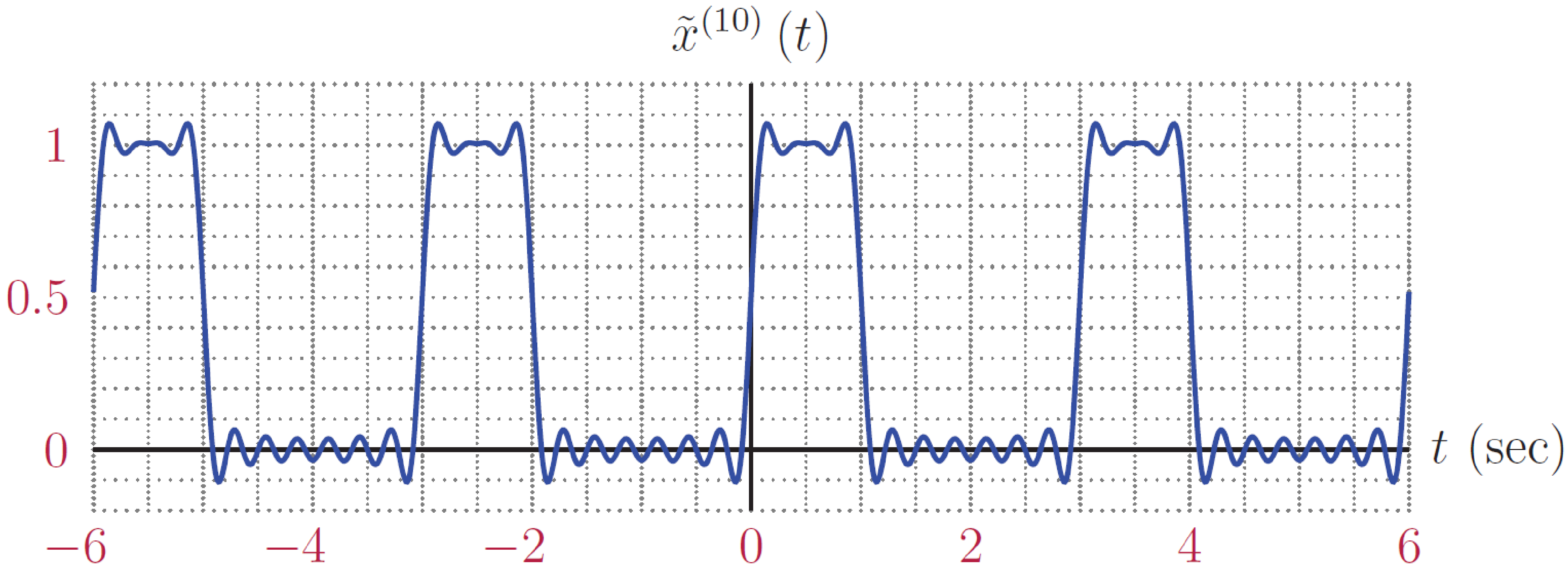
$$+ 0.4775 \sin(2\pi t/3) + 0.2387 \sin(4\pi t/3) + 0.1194 \sin(8\pi t/3)$$

A similar but lengthier expression can be written for the case $m = 10$ which we will skip to save space. Fig. 4.6 shows two approximations to the original pulse train using the first 4 and 10 harmonics respectively.

Example 4.2 – Solution



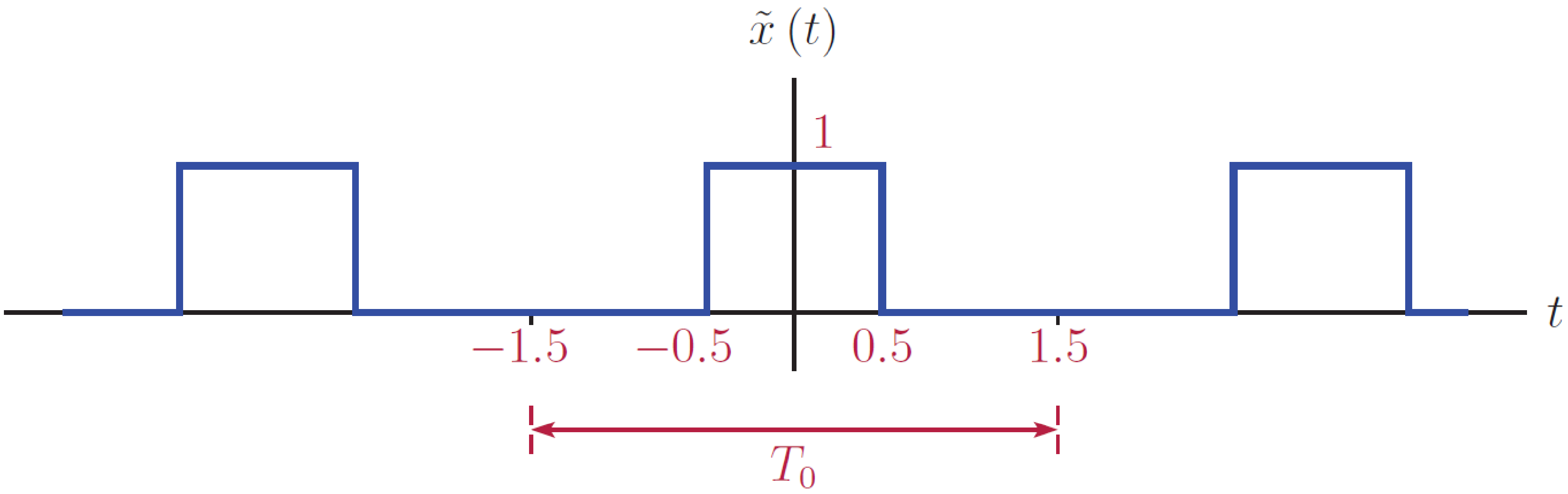
Example 4.2 – Solution



Example 4.3

Example 4.3: Periodic pulse train revisited

Determine the TFS coefficients for the periodic pulse train shown in Fig. 4.7.



Example 4.3 – Solution

Solution: This is essentially the same pulse train we have used in Example 4.1 with one minor difference: The signal is shifted in the time domain so that the main pulse is centered around the time origin $t = 0$. As a consequence, the resulting signal is an even function of time, that is, it has the property $\tilde{x}(-t) = \tilde{x}(t)$ for $-\infty < t < \infty$.

Let us take one period of the signal to extend from $t_0 = -1.5$ to $t_0 + T_0 = 1.5$ seconds. Applying Eqn. (4.43) with $t_0 = -1.5$ and $T_0 = 3$ seconds, we have

$$a_0 = \frac{1}{3} \int_{t=-0.5}^{0.5} (1)\, dt = \frac{1}{3}$$

Example 4.3 – Solution

Using Eqn. (4.41) yields

$$a_k = \frac{2}{3}\int_{-0.5}^{0.5} (1)\,\cos\left(2\pi k t/3\right)\,dt = \frac{2\,\sin\left(2\pi k/3\right)}{\pi k}$$

and using Eqn. (4.42)

$$b_k = \frac{2}{3}\int_{-0.5}^{0.5} (1)\,\sin\left(2\pi k t/3\right)\,dt = 0$$

Thus, $\tilde{x}(t)$ can be written as

$$\tilde{x}(t) = \frac{1}{3} + \sum_{k=1}^{\infty}\left(\frac{2\,\sin\left(\pi k/3\right)}{\pi k}\right)\cos\left(k\omega_0 t\right)$$

Problem 4.2

4.2. Consider the pulse train shown in Fig. P.4.2.

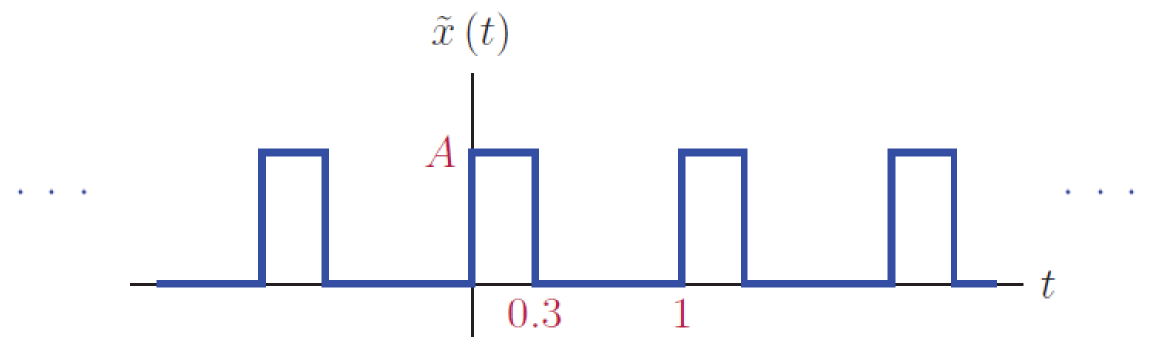


Figure P. 4.2

a. Determine the fundamental period T_0 and the fundamental frequency ω_0 for the signal.

b. Using the technique described in Section 4.2 find an approximation to $\tilde{x}(t)$ in the form

$$\tilde{x}^{(1)}(t) \approx a_0 + a_1 \cos(\omega_0 t) + b_1 \sin(\omega_0 t)$$

Determine the optimum coefficients a_0, a_1 and b_1.

Problem 4.2 – Solution

a. The fundamental period is $T_0 = 1$ second which corresponds to a fundamental frequency of $f_0 = 1$ Hz or $\omega_0 = 2\pi$ rad/s.

b.

$$a_0 = \int_0^1 \tilde{x}(t)\, dt = 0.3\, A$$

$$a_1 = 2\int_{-1}^{1} \tilde{x}(t) \cos(2\pi t)\, dt = \frac{A \sin(0.6\pi)}{\pi}$$

$$b_1 = 2\int_0^1 \tilde{x}(t) \sin(2\pi t)\, dt = \frac{A\left[1 - \cos(0.6\pi)\right]}{\pi}$$

Problem 4.3

4.3. Consider again the pulse train shown in Fig. P.4.2. Using the technique described in Section 4.2 find an approximation to $\tilde{x}(t)$ in the form

$$\tilde{x}^{(2)}(t) \approx a_0 + a_1 \cos(\omega_0 t) + b_1 \sin(\omega_0 t) + a_2 \cos(2\omega_0 t) + b_2 \sin(2\omega_0 t)$$

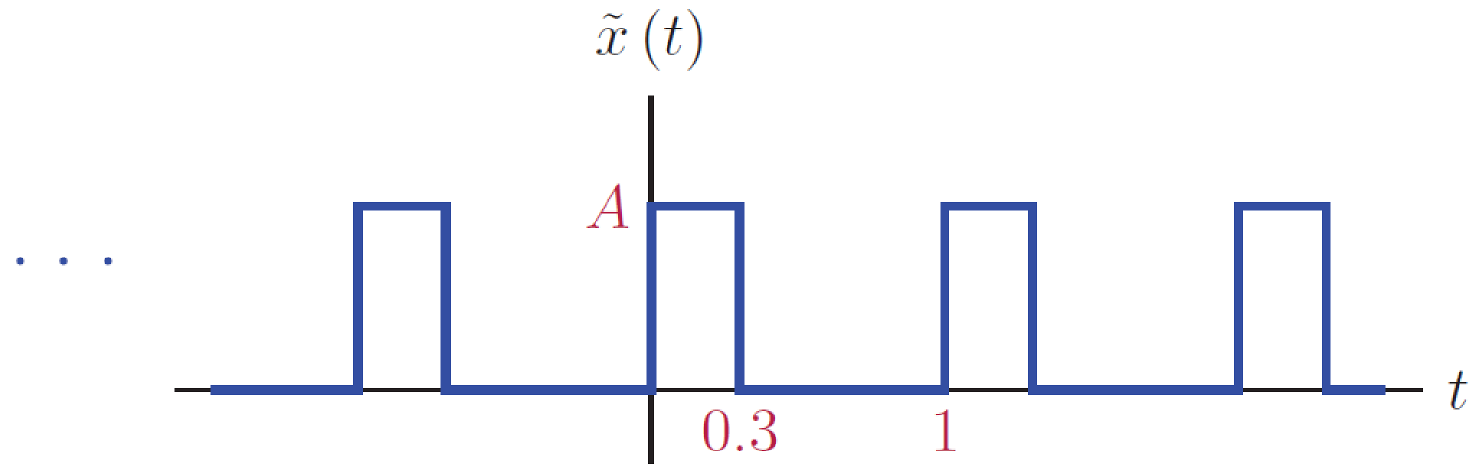


Figure P. 4.2

Problem 4.3 – Solution

$$a_0 = \int_0^1 \tilde{x}(t)\, dt = 0.3\, A$$

$$a_1 = 2\int_0^1 \tilde{x}(t) \cos(2\pi t)\, dt = \frac{A \sin(0.6\pi)}{\pi}$$

$$b_1 = 2\int_0^1 \tilde{x}(t) \sin(2\pi t)\, dt = \frac{A\left[1 - \cos(0.6\pi)\right]}{\pi}$$

$$a_2 = 2\int_0^1 \tilde{x}(t) \cos(4\pi t)\, dt = \frac{A \sin(1.2\pi)}{2\pi}$$

$$b_2 = 2\int_0^1 \tilde{x}(t) \sin(4\pi t)\, dt = \frac{A\left[1 - \cos(1.2\pi)\right]}{2\pi}$$

Problem 4.5

4.5. Consider the pulse train $\tilde{x}(t)$ shown in Fig. P.4.5.

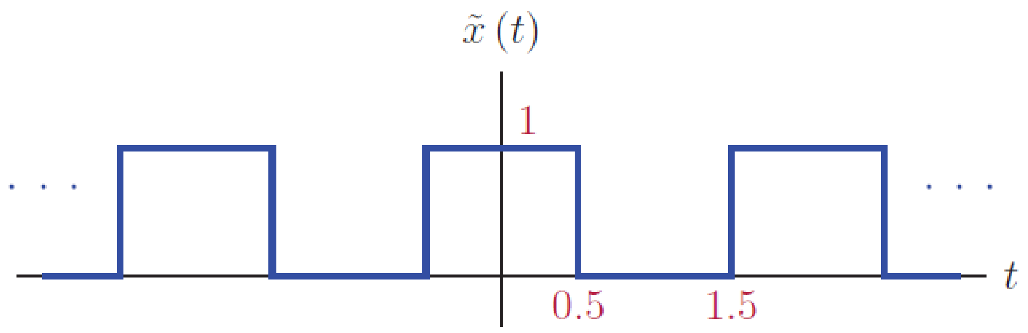


Figure P. 4.5

a. Determine the fundamental period T_0 and the fundamental frequency ω_0 for the signal.

b. Using the approach followed in Section 4.2 determine the coefficients of the approximation

$$\tilde{x}^{(2)}(t) = a_0 + a_1 \cos(\omega_0 t) + a_2 \cos(2\omega_0 t)$$

to the signal $\tilde{x}(t)$ that results in the minimum mean-squared error.

Problem 4.5 – Solution

a. The fundamental period is $T_0 = 2$ seconds which corresponds to a fundamental frequency of $f_0 = 1/2$ Hz or $\omega_0 = \pi$ rad/s.

b.

$$a_0 = \frac{1}{2}\int_{-1}^{1} \tilde{x}(t)\, dt = \frac{1}{2}$$

$$a_1 = \int_{0}^{1} \tilde{x}(t)\cos(\pi t)\, dt = \frac{2}{\pi}$$

$$a_2 = \int_{-1}^{1} \tilde{x}(t)\cos(2\pi t)\, dt = 0$$

Exponential Fourier Series (EFS)

- Fourier series representation of the periodic signal $\tilde{x}(t)$ can also be written in alternative forms.
- Consider the use of complex exponentials as basis functions so that the signal $\tilde{x}(t)$ is expressed as

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} c_k \, e^{jk\omega_0 t}$$

- This is referred to as the exponential Fourier series (EFS) representation of the periodic signal.

Euler’s Formula

$$\cos(-x) = \cos(x)$$
$$\sin(-x) = -\sin(x)$$

$$e^{jx} = \cos(x) + j\sin(x)$$
$$e^{-jx} = \cos(-x) + j\sin(-x)$$
$$e^{-jx} = \cos(x) - j\sin(x)$$

$$e^{jx} + e^{-jx} = 2\cos(x)$$
$$\cos(x) = \frac{1}{2}e^{jx} + \frac{1}{2}e^{-jx}$$

Exponential Fourier Series (EFS)

Exponential Fourier series (EFS):

1. Synthesis equation:

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} c_k \, e^{jk\omega_0 t}$$

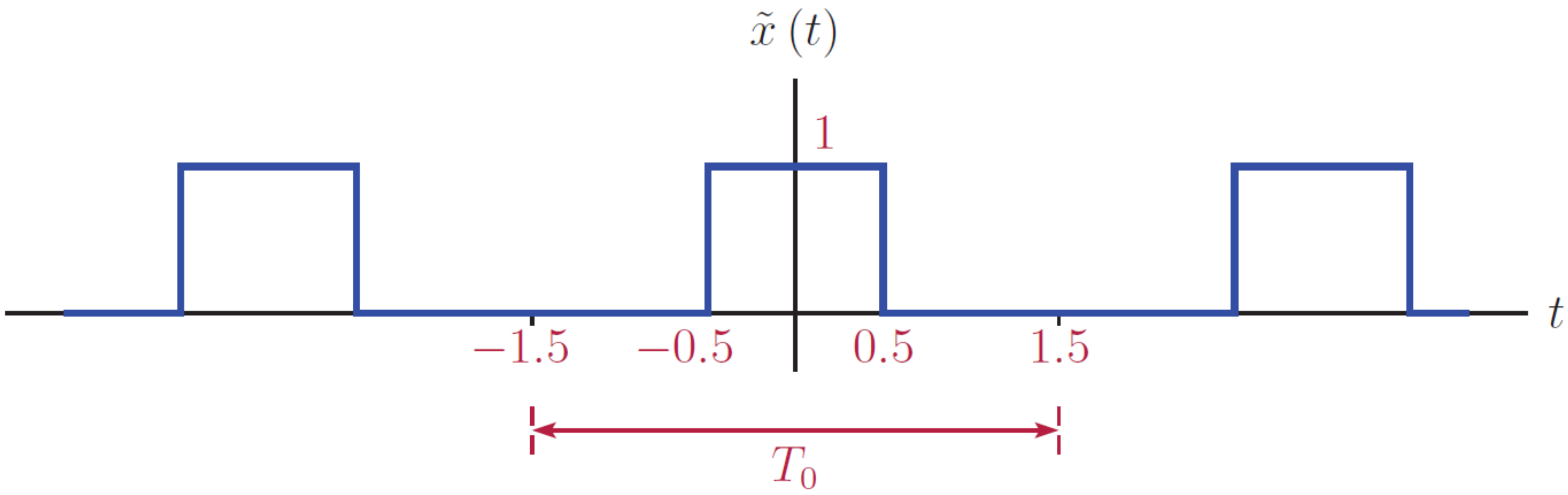
2. Analysis equation:

$$c_k = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} \tilde{x}(t) \, e^{-jk\omega_0 t} \, dt$$

Example 4.5

Example 4.5: Exponential Fourier series for periodic pulse train

Determine the EFS coefficients of the signal $\tilde{x}(t)$ of Example 4.3, shown in Fig. 4.7, through direct application of Eqn. (4.72).



Example 4.5 – Solution

$$c_k = \frac{1}{3}\int_{-0.5}^{0.5}(1)e^{-j\frac{2\pi k}{3}t}dt = \frac{1}{3}.\frac{3}{-j2\pi k}e^{-j\frac{2\pi k}{3}t}\Big|_{-0.5}^{0.5} = \frac{1}{-j2\pi k}e^{-j\frac{2\pi k}{3}t}\Big|_{-0.5}^{0.5}$$

$$= \frac{1}{-j2\pi k}\left[\cos\left(-\frac{2\pi k}{3}t\right) + j\sin\left(-\frac{2\pi k}{3}t\right)\right]_{-0.5}^{0.5}$$

$$= \frac{1}{-j2\pi k}\left[\cos\left(\frac{2\pi k}{3}t\right) - j\sin\left(\frac{2\pi k}{3}t\right)\right]_{-0.5}^{0.5}$$

$$= \frac{1}{-j2\pi k}\left[0 - j\sin\left(\frac{2\pi k}{3}t\right)\right]_{-0.5}^{0.5} = \frac{1}{2\pi k}\left[\sin\left(\frac{2\pi k}{3}t\right)\right]_{-0.5}^{0.5}$$

$$= \frac{1}{2\pi k}\left[\sin\left(\frac{\pi k}{3}\right) - \sin\left(-\frac{\pi k}{3}\right)\right] = \frac{1}{2\pi k}\left[\sin\left(\frac{\pi k}{3}\right) + \sin\left(\frac{\pi k}{3}\right)\right]$$

$$= \frac{1}{2\pi k}\left[2\sin\left(\frac{\pi k}{3}\right)\right] = \frac{1}{\pi k}\sin\left(\frac{\pi k}{3}\right)$$

Example 4.5 – Solution

Solution: Using Eqn. (4.72) with $t_0 = -1.5$ s and $T_0 = 3$ s, we obtain

$$c_k = \frac{1}{3} \int_{-0.5}^{0.5} (1)\, e^{-j2\pi kt/3}\, dt = \frac{\sin(\pi k/3)}{\pi k}$$

The signal $\tilde{x}(t)$ can be expressed in terms of complex exponential basis functions as

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} \left(\frac{\sin(\pi k/3)}{\pi k} \right) e^{j2\pi kt/3}$$

Example 4.5 – Solution

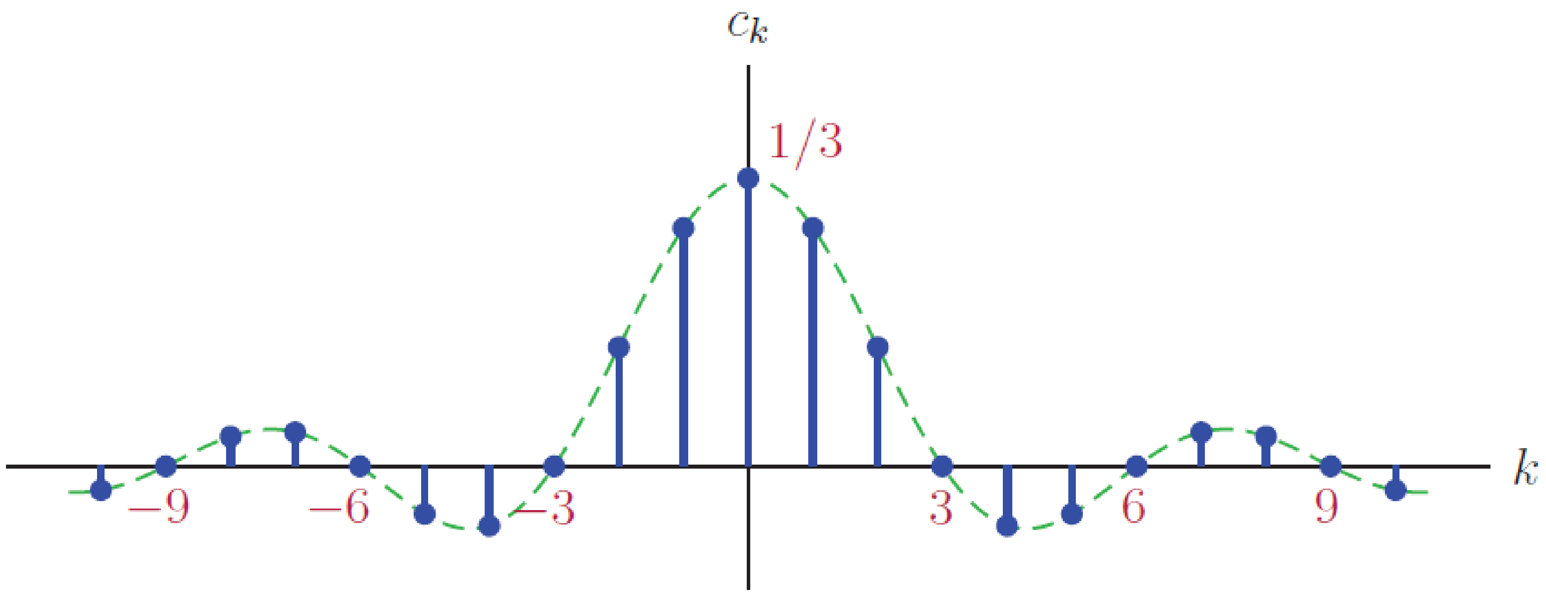


Figure 4.11 – The line spectrum for the pulse train of Example 4.5.

Example 4.9

Example 4.9: Spectrum of multi-tone signal

Determine the EFS coefficients and graph the line spectrum for the multi-tone signal

$$\tilde{x}(t) = \cos\left(2\pi\left[10f_0\right]t\right) + 0.8\,\cos\left(2\pi f_0 t\right)\,\cos\left(2\pi\left[10f_0\right]t\right)$$

shown in Fig. 4.19.

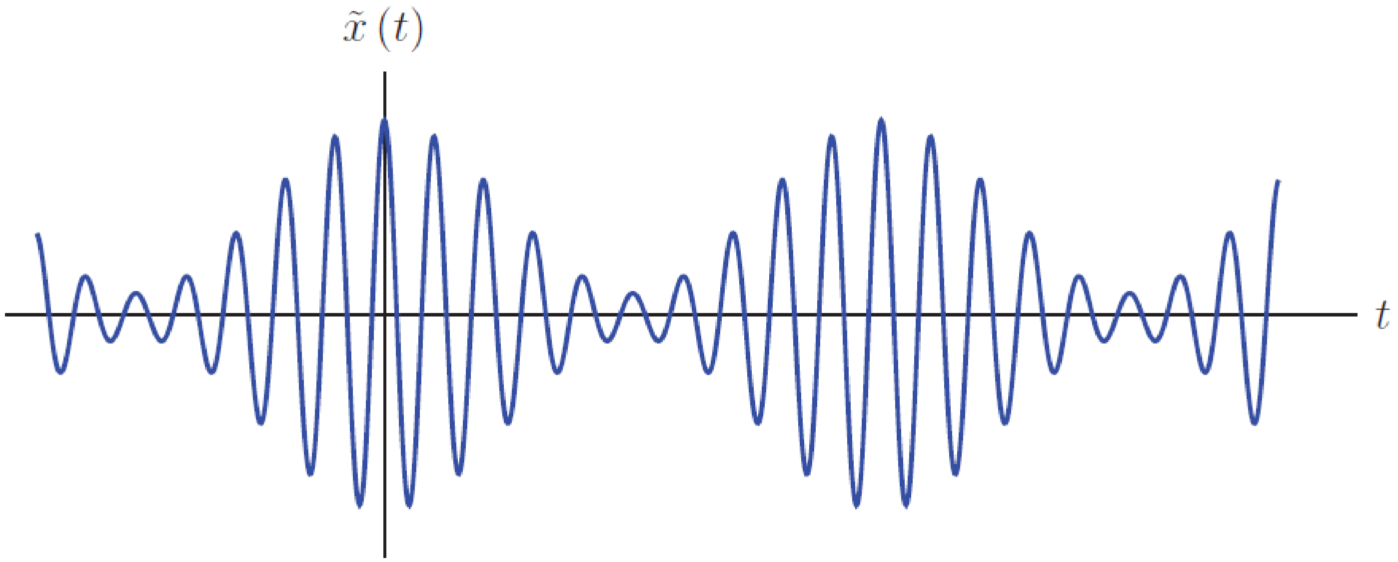


Figure 4.19 – Multi-tone signal of Example 4.9.

Example 4.9 – Solution

$$\tilde{x}(t) = \cos(2\pi 10 f_0 t) + 0.8\cos(2\pi f_0 t)\cos(2\pi 10 f_0 t)$$

Using $\cos(a)\cos(b) = 0.5\cos(a+b) + 0.5\cos(a-b)$

$$\tilde{x}(t) = \cos(2\pi 10 f_0 t) + 0.4\cos(22\pi f_0 t) + 0.4\cos(18\pi f_0 t)$$

$$\tilde{x}(t) = \cos(2\pi 10 f_0 t) + 0.4\cos(2\pi 11 f_0 t) + 0.4\cos(2\pi 9 f_0 t)$$

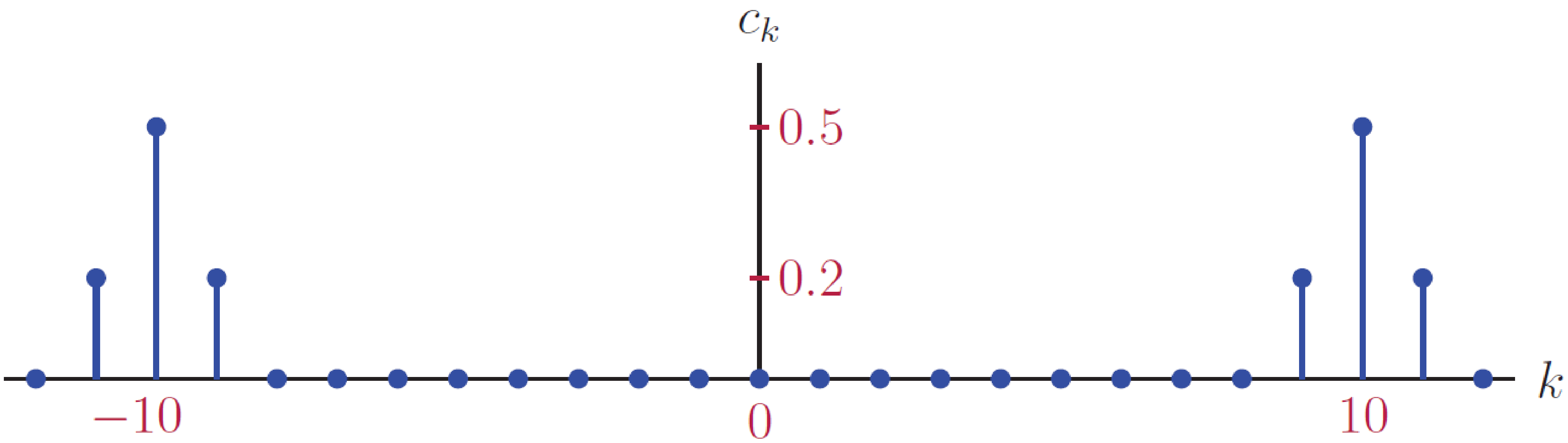
Applying $\cos(x) = 0.5e^{jx} + 0.5e^{-jx}$

$$\tilde{x}(t) = 0.5e^{j2\pi 10 f_0 t} + 0.5e^{-j2\pi 10 f_0 t} + 0.2e^{j2\pi 11 f_0 t} + 0.2e^{-j2\pi 11 f_0 t} + 0.2e^{j2\pi 9 f_0 t} + 0.2e^{-j2\pi 9 f_0 t}$$

Example 4.9 – Solution

$$\tilde{x}(t) = 0.5e^{j2\pi 10 f_0 t} + 0.5e^{-j2\pi 10 f_0 t} + 0.2e^{j2\pi 11 f_0 t} + 0.2e^{-j2\pi 11 f_0 t} + 0.2e^{j2\pi 9 f_0 t} + 0.2e^{-j2\pi 9 f_0 t}$$

$$\begin{aligned} c_9 &= c_{-9} = 0.2, \\ c_{10} &= c_{-10} = 0.5, \\ c_{11} &= c_{-11} = 0.2 \end{aligned}$$



Compact Fourier Series (CFS)

- Yet another form of the Fourier series representation of a periodic signal is the compact Fourier series (CFS) expressed as

$$\tilde{x}(t) = d_0 + \sum_{k=1}^{\infty} d_k \cos(k\omega_0 t + \phi_k)$$

Sinusoids

- Sinusoids with time varying frequency generate interesting sounds.
- For instance, consider the following signal:

$$x(t) = A\cos\left(\omega t + \frac{t^2}{4}\right)$$

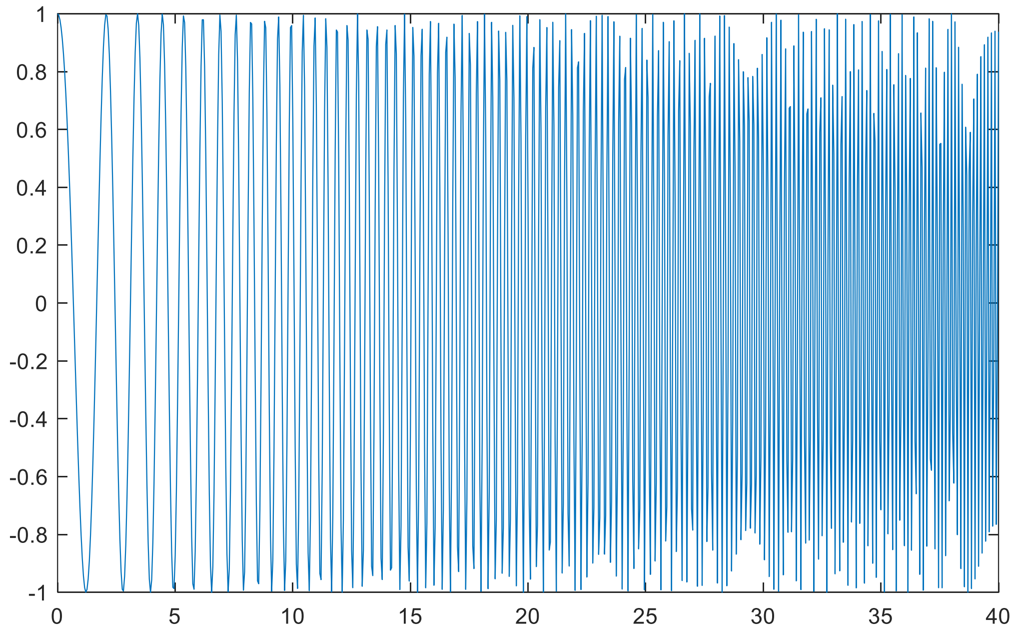
Let $A = 1$ and $\omega = 2$.

Use MATLAB to plot this signal for $0 \le t \le 40$ in steps of 0.05.

Use sound to listen to the signal.

Sinusoids

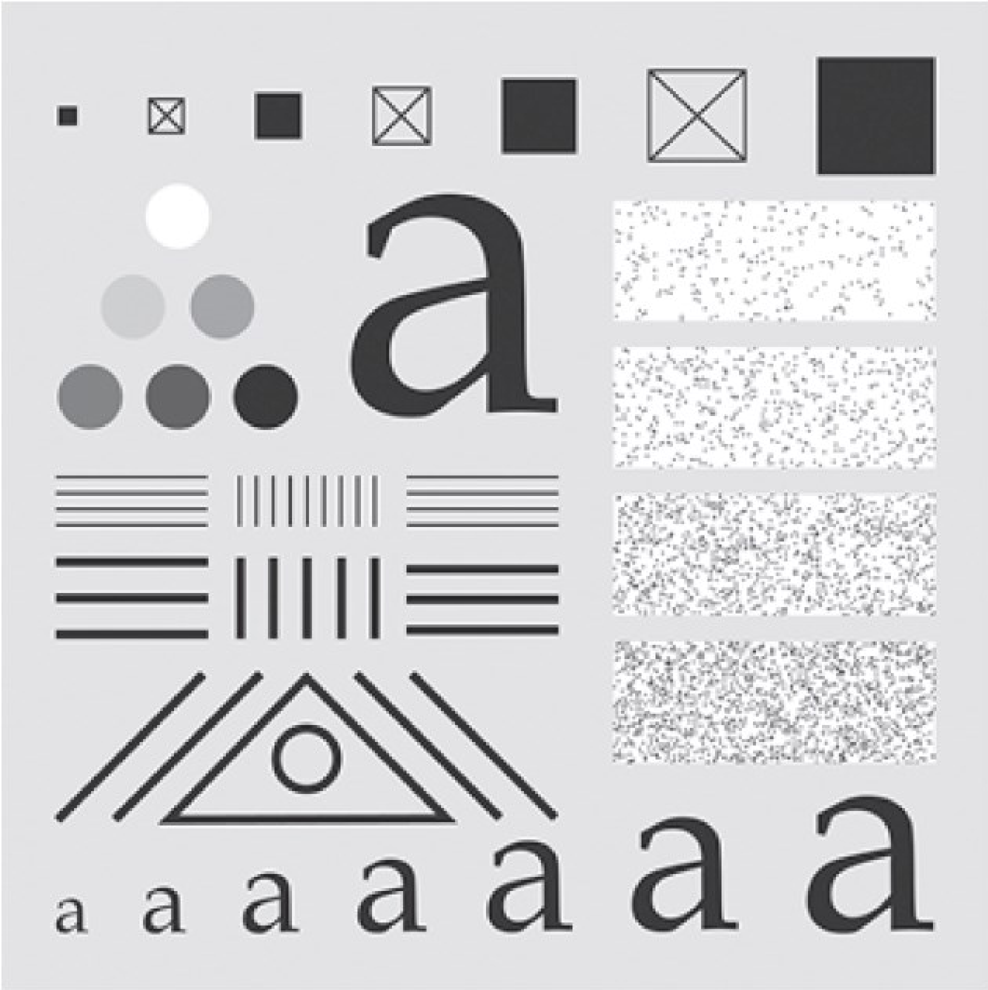
```
A = 1;
w = 2;
x = @(t) A*cos(w*t + t.^2/4);

t = 0:0.05:40;
xt = x(t);
plot(t, xt);

sound(xt);
```

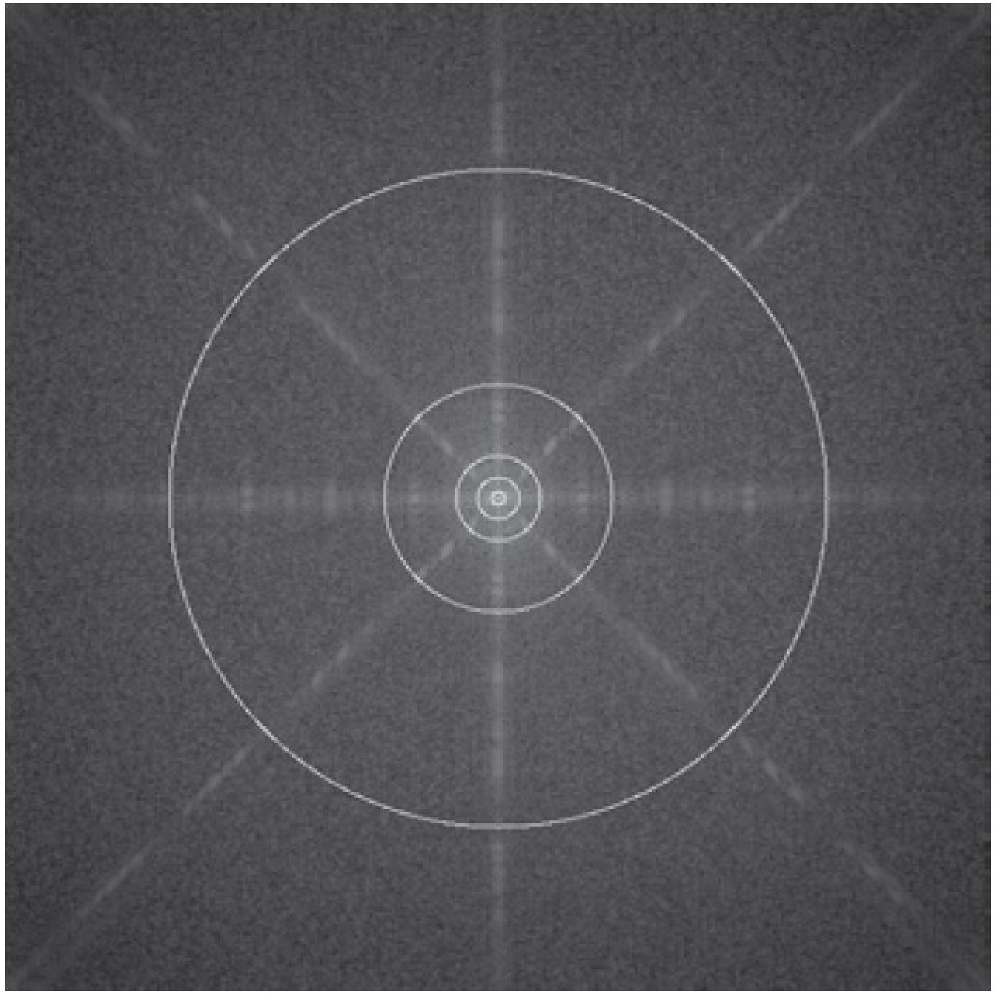


Interactive Demos

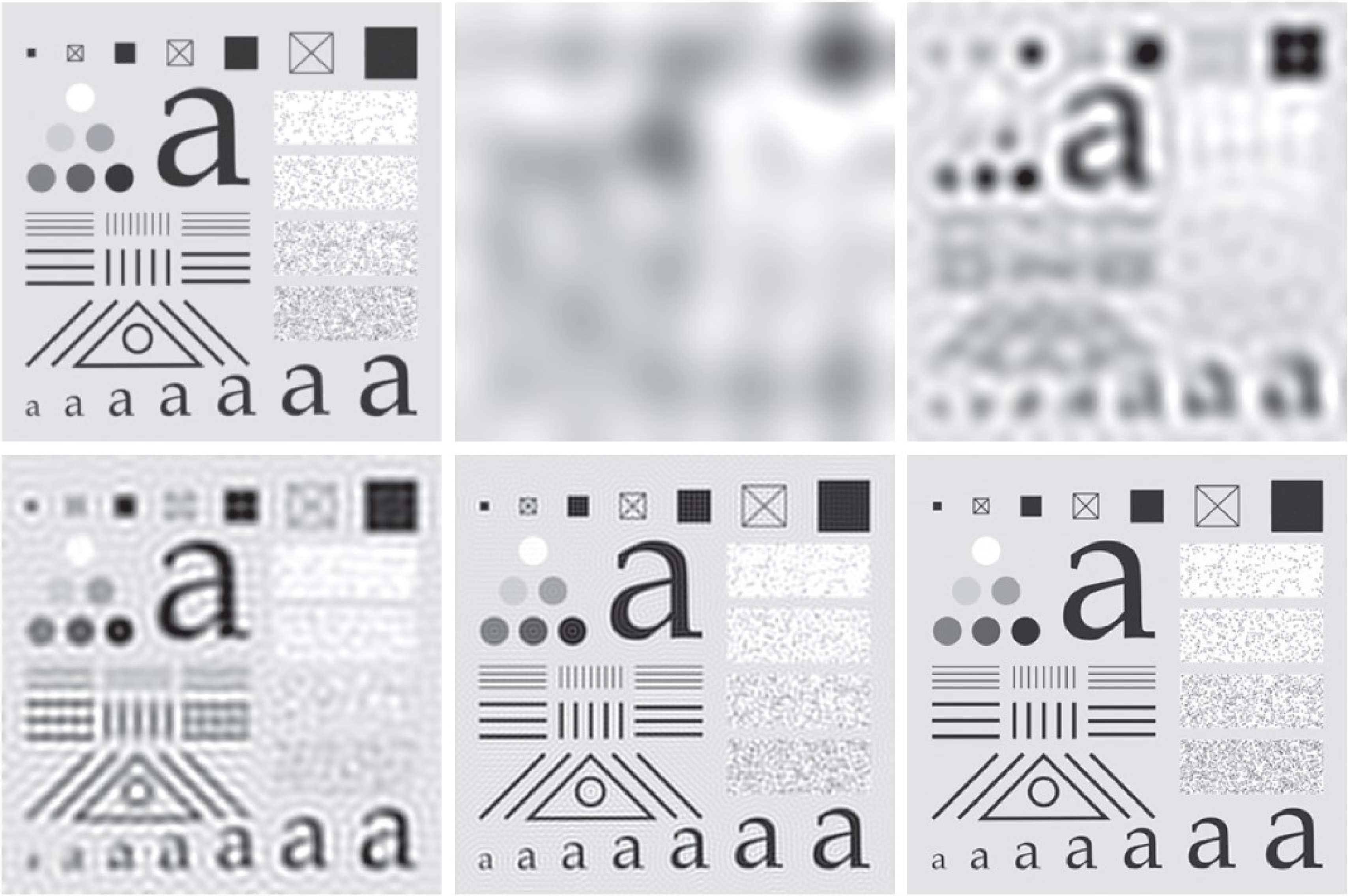
```
>> appr_demo1
>> tfs_demo1
>> tfs_demo2
>> tfs_demo3
>> efs_demo1
```

Filtering in the Frequency Domain

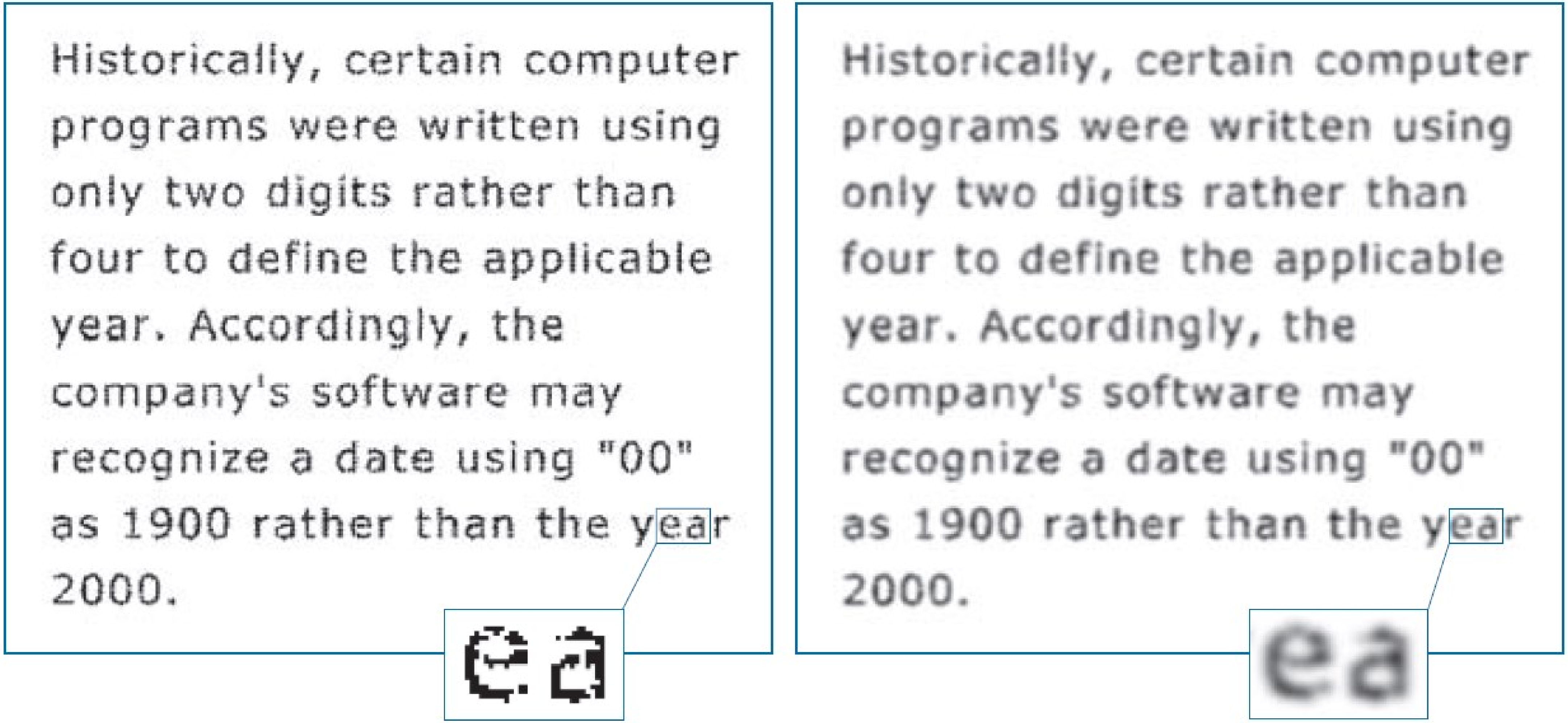




Filtering in the Frequency Domain: Lowpass Filters



Filtering in the Frequency Domain: Lowpass Filters



Filtering in the Frequency Domain: Highpass Filters

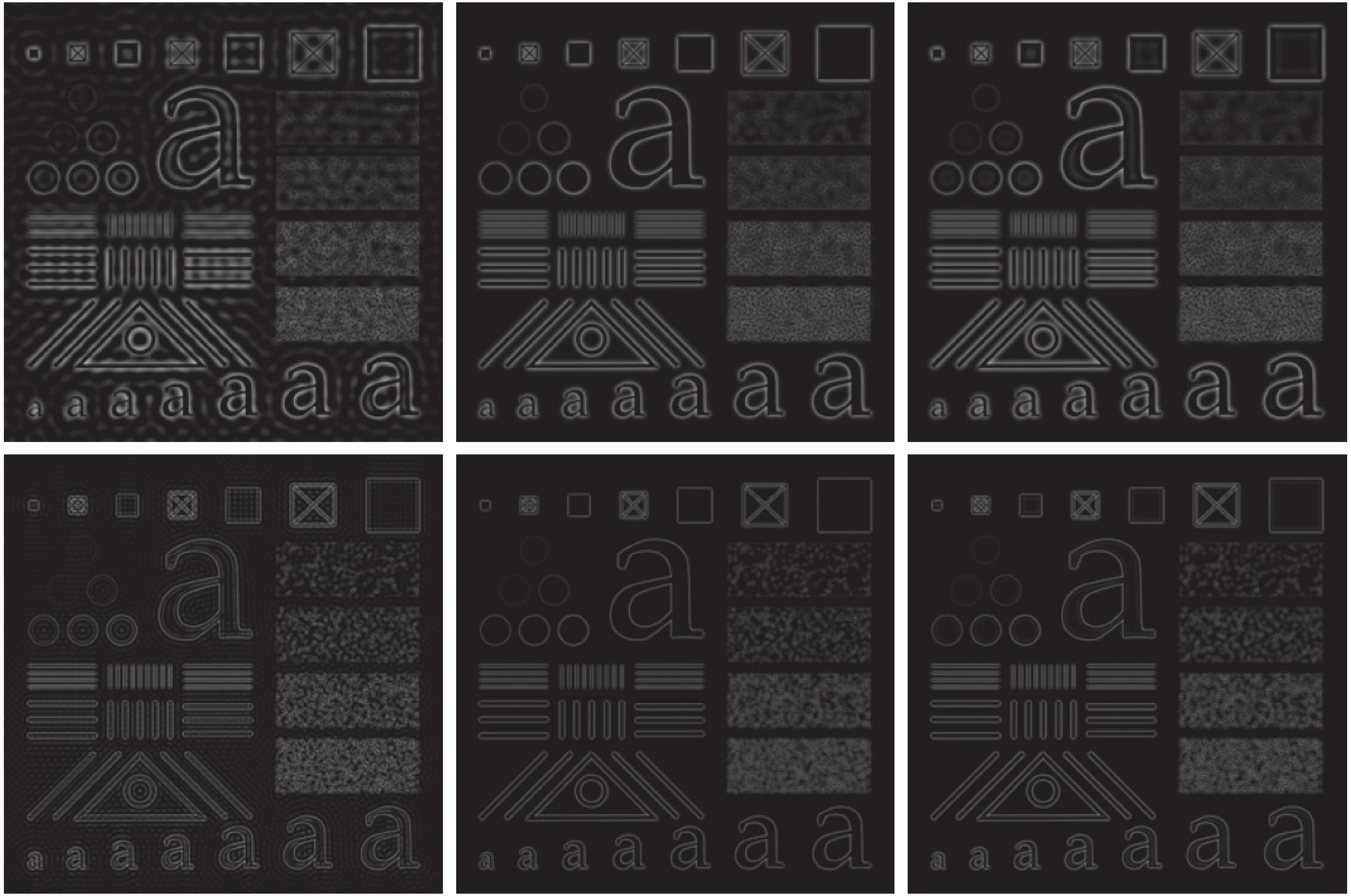
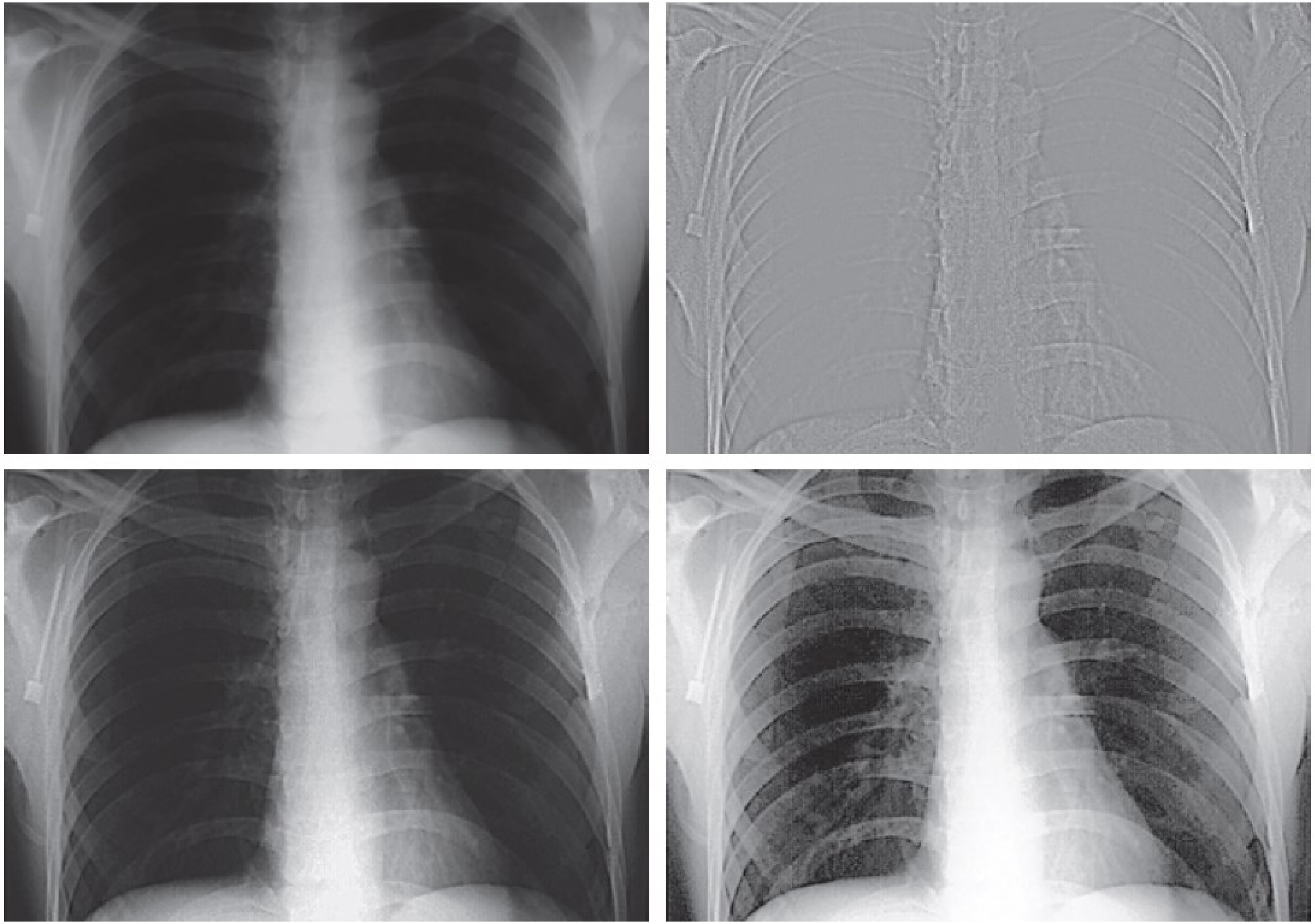


Image Enhancement Using the Laplacian in the Frequency Domain

Image Enhancement in the Frequency Domain



Periodic Noise Reduction Using Frequency Domain